Webology (ISSN: 1735-188X)
Volume 17, Number 2, 2020

Steiner Domination In Line And Jump Fuzzy Graphs

[1]Priscilla Pacifica G , [2]Jenit Ajitha J

[1]Assistant Professor, St.Marys' College(Autonomous),Thoothukudi,Tamilnadu,

[2]Research Scholar(Reg No:19222212092011), St.Marys' College(Autonomous), Thoothukudi

(Affiliated to Manonmaniam Sundaranar University, Tirunelveli, Tamilnadu, India.

Abstract—Line graph L(G) of a graph G is acquired by converting the arcs of G into nodes of the L(G) and connecting the nodes of L(G) only if the corresponding arcs are incident with the same node. The jump graph J(G) is the complement of L(G). In this article bounds on steiner domination numbers of line fuzzy graphs and jump fuzzy graphs are obtained.

Keywords : fuzzy steiner domination, line fuzzy graphs, jump fuzzy graphs.
AMS Subject Classification 2010 : 05C72, 05C69, 51E10

1. Introduction

Rosenfeld launched fuzzy graph theory which has its applications in diverse fields. In particular fuzzy topologies are used in circuit designing and fuzzy steiner distance and domination have applications in routing problems. In engineering field steiner trees have applications in network routing, wireless communications and VLSI design. Various fuzzy graph theoretic concepts has been studied from [6] and [7]. In [1] and [2] the authors described about domination in fuzzy graphs. Steiner domination in crisp graphs was studied from [3], [4] and [5]. A steiner set of a fuzzy graph (V, σ, μ) is a set of nodes S such that any node in G lies in some steiner tree of G. A steiner dominating set of G is a set of nodes which is both steiner set as well as dominating set. The minimum fuzzy cardinality of a minimal fuzzy Steiner dominating set is called fuzzy Steiner dominating number denoted by γ^{fs} and the maximum fuzzy cardinality of a minimal fuzzy Steiner dominating set is called upper fuzzy Steiner dominating number denoted by Γ^{fs}. Here we acquire some bounds on steiner domination numbers of line fuzzy graphs and jump fuzzy graphs.

2. Steiner Domination in Line fuzzy and Jump fuzzy graphs

2.1 Definition

For any fuzzy graph G(V, σ, μ), the line fuzzy graph $G'(V', \sigma', \mu') = L(G)$ is defined as follows. The arcs of G are transformed into the nodes of L(G). The membership values of nodes and arcs are given as follows.

i) $\sigma'(uv) = \mu(u, v)$ where uv is an arc in G.

ii) $\mu'(x, y) = \mu(u_1, v_1) \wedge \mu(u_2, v_2)$ where $x = u_1v_1$ and $y = u_2v_2$ are arcs in G.

The line fuzzy graph of any fuzzy graph is always an effective strong arc fuzzy graph.

2.2 Definition

The complement fuzzy graph of line fuzzy graph L(G) is termed as the jump fuzzy graph of G.(i.e) $L(G)^c = J(G) = G''(V'', \sigma'', \mu'')$. The membership values of the jump graph are as follows. $\sigma'' = \sigma'$ and $\mu''(x, y) = \sigma'(x) \wedge \sigma'(y) - \mu'(x, y)$.

2.3 Theorem

The steiner domination number of a wheel fuzzy graph with n nodes denoted by W_n^f is given by

$$\gamma^{fs}(L(W_n^f)) \leq \begin{cases} \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ if n is odd} \\ \frac{n}{2} \qquad \text{n is even} \end{cases}$$

Proof :

The line fuzzy graph of W_n^f is a strong fuzzy graph with $2(n-1)$ nodes. Let $e_1, e_2, \dots, e_{n-1}$ be the arcs of W_n^f connecting the centre node and $e_n, e_{n+1}, \dots, e_{2(n-1)}$ be the arcs along the apex (i.e) along the fuzzy cycle. In the line fuzzy graph of W_n^f, the induced subgraph of the nodes $U = \{e_1, e_2, \dots, e_{n-1}\}$ is a complete fuzzy graph and the induced subgraph of the nodes $V = \{e_n, e_{n+1}, \dots, e_{2(n-1)}\}$ is a strong fuzzy cycle. Each node from V is adjacent to exactly two nodes from U. There is no extreme node. Label the nodes in V such that e_n is adjacent to e_1& e_2, e_{n+1} is adjacent to e_2& e_3 and so on as given below in the figure.

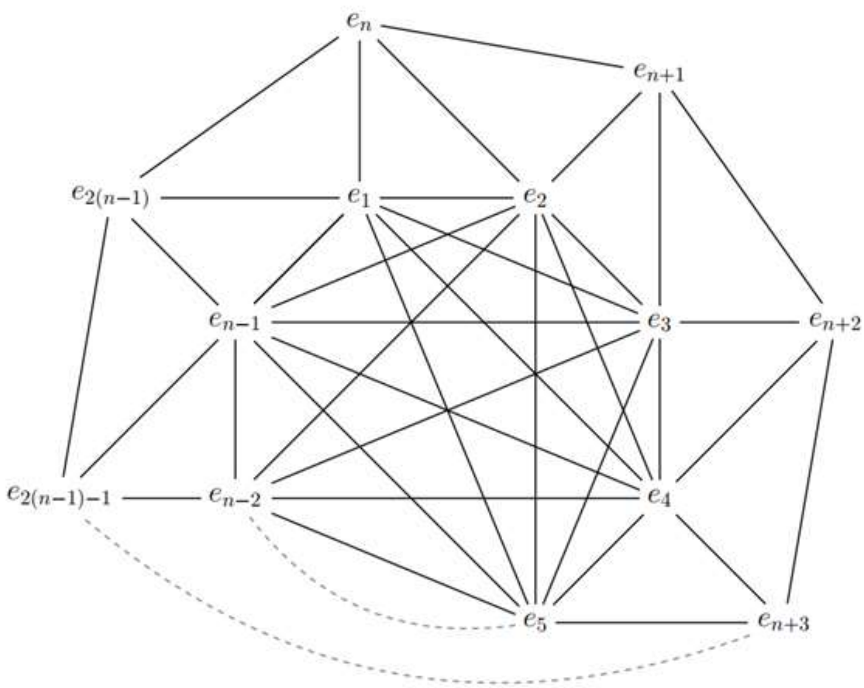

Fig 1

For $n = 4$, the set of any two non-adjacent nodes forms a minimum steiner dominating set. Hence $\gamma^{fs}(L(W_4^f)) \leq 2$.

For $n = 5$, the set of nodes $\{e_1, e_6, e_7\}$ is a minimum steiner dominating set. Therefore $\gamma^{fs}(L(W_5^f)) \leq 3$.

Now assume that $n > 5$. Since the induced subgraph of U is a complete fuzzy graph, any subset of U cannot be a steiner set. Also any steiner tree of V does not contain any of the node from U. Thus any subset of V does not form a steiner set. Now there are two cases here.

Case (i) n is even

Consider the set of nodes $S = \{e_1, e_{n+1}, e_{n+3}, e_{n+5}, \ldots. e_{2n-3}\}$. It can be observed that S is a steiner dominating set with $\frac{n}{2}$ nodes. If e_1 is omitted from S, then the other nodes of $S - \{e_1\}$ are along the fuzzy cycle and hence any steiner tree of $S - \{e_1\}$ contains no node from U. If any other node from S, $e_{n+(2i+1)}$ other than e_1 is removed, then it does not belong to any steiner tree of $S - \{e_{n+(2i+1)}\}$. Thus S is a minimal steiner dominating set and it is the minimum.

Hence $\gamma^{fs}(L(W_n^f)) \leq \frac{n}{2}$.

Case (ii) n is odd

Subcase (i) $n \equiv 1(mod\ 4)$

Let $S' = \{e_1, e_{n+1}, e_{n+3} \ldots, e_{n+\left(\left\lfloor\frac{n}{2}\right\rfloor-1\right)}, e_{n+\left(\left\lfloor\frac{n}{2}\right\rfloor\right)}, e_{n+\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)} \ldots \ldots e_{2(n-1)-1}\}$. It can be verified that S' is a minimal steiner dominating set with and by similar argument in case (i) it is the minimum steiner dominating set with $\left\lfloor \frac{n}{2} \right\rfloor + 1$ nodes.

Subcase (ii) $n \equiv 3(mod\ 4)$

For this case

$S'' = \{e_1, e_{\frac{n+1}{2}}, e_{n+1}, e_{n+3}, \ldots\ldots, e_{n+\left(\left\lfloor\frac{n}{2}\right\rfloor-2\right)}, e_{n+\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}, e_{n+\left(\left\lfloor\frac{n}{2}\right\rfloor+3\right)}, \ldots.\, e_{2(n-1)-1}$ is a minimum steiner dominating set with $\left\lfloor \frac{n}{2} \right\rfloor + 1$nodes. Thus $\gamma^{fs}(L(W_n^f)) \leq \left\lfloor \frac{n}{2} \right\rfloor + 1$.

2.4 Results and Observations

i) Line graph of a path fuzzy graph with ‘n’ nodes is a strong path fuzzy graph with ‘n-1’ nodes. Hence $\gamma^{fs}(L(P_n^f)) \leq \gamma^{fs}(L(P_{n-1}^f))$.

ii) Line graph of a fuzzy cycle with ‘n’ nodes is a strong fuzzy cycle with ‘’n’ nodes. Jump graph of fuzzy cycle is a star fuzzy graph.

iii) Line graph of a star fuzzy graph $K_{1,n}^f$ is a complete fuzzy graph with ‘n’ nodes. Hence $\gamma^{fs}(L(K_{1,n}^f)) \leq n$. Jump graph of $K_{1,n}^f$ is complement of $L(K_{1,n}^f)$ which is the empty graph with n nodes.

Thus $\gamma^{fs}\left(J\left(K_{1,n}^f\right)\right) = \gamma^{fs}(L(K_{1,n}^f)) \leq n$.

2.5 Theorem

The steiner domination number of jump graph of a path fuzzy graph with ‘n’ nodes is given by $\gamma^{fs}\left(J\left(P_n^f\right)\right) \leq 3$.

Proof :

The jump graph of a path fuzzy graph with ‘n’ nodes is the complement of the strong path fuzzy graph with ‘n-1’ nodes.

For $n = 2, J\left(P_2^f\right)$ is nothing but K_1^f and so $\gamma^{fs}\left(J\left(P_2^f\right)\right) \leq 1$.

For $n = 3, J\left(P_3^f\right)$ is the empty fuzzy graph with two nodes. So $\gamma^{fs}\left(J\left(P_3^f\right)\right) \leq 2$.

For $n = 4, J\left(P_4^f\right)$ is a disconnected fuzzy graph with one K_1^f-component and one K_2^f- component. Thus $\gamma^{fs}\left(J\left(P_4^f\right)\right) \leq 3$.

Now assume that $n > 4$. Let $u_1, u_2, \ldots., u_n$ be the nodes and $e_1, e_2, \ldots., e_{n-1}$ are arcs of P_n^f such that $e_i = u_i u_{i+1}$ for $i = 1,2, \ldots., n-1$. Then $e_1, e_2, \ldots., e_{n-1}$ are the nodes of $L\left(P_n^f\right)$ which is a path fuzzy graph with ‘n-1’ nodes.

If $n = 5$, the set of nodes $\{e_2, e_3\}$ forms a minimum steiner dominating set which are diametral nodes. So $\gamma^{fs}\left(J\left(P_5^f\right)\right) \leq 2$.

If $n > 5$, the set of nodes $\{e_1, e_2, e_3\}$ is a minimum steiner dominating set because if we choose any two non-adjacent nodes it does not form a steiner set.

Hence the theorem.

2.6 Theorem

$\gamma^{fs}\left(J(C_n^f)\right) \leq 4$ where C_n^f is a cycle fuzzy graph with 'n' nodes.

Proof :

Let $e_1, e_2, \ldots, e_n$ be the arcs of C_n^f which are the nodes of $L(C_n^f)$ and $J(C_n^f)$ which is the complement of $L(C_n^f)$. If $n = 3$, $J(C_n^f)$ is a disconnected fuzzy graph with three K_1^f-components. Hence $\gamma^{fs}\left(J(C_3^f)\right) \leq 3$.

For $n = 4$, $J(C_n^f)$ is a disconnected fuzzy graph with two K_2^f-components. Therefore $\gamma^{fs}\left(J(C_4^f)\right) \leq 4$. If assume that $n > 4$, as in the proof of previous theorem $\{e_1, e_2, e_3\}$ is a minimum steiner dominating set. Hence the result.

2.7 Theorem

$\gamma^{fs}\left(J(W_n^f)\right) \leq n - 1$ for $n > 4$ where W_n^f is a wheel fuzzy graph with 'n' nodes.

Proof :

The jump graph of a wheel fuzzy graph W_n^f which is the complement of line graph of W_n^f has $2(n-1)$ arcs. Let $e_1, e_2, \ldots, e_{2(n-1)}$ be the nodes of $J(W_n^f)$.

If $n = 4$, then $J(W_n^f)$ is a disconnected fuzzy graph with three K_2^f-components. Hence $\gamma^{fs}\left(J(W_4^f)\right) = p$ where p is the order of $J(W_4^f)$. Now assume that $n > 4$. In the wheel fuzzy graph, W_n^f let $U = \{e_1, e_2, \ldots, e_{n-1}\}$ be the set of arcs connecting the apex nodes to the centre nodes and $V = \{e_n, e_{n+1}, \ldots, e_{2(n-1)}\}$ be the set of nodes on the apex and labelled such that e_n is adjacent to e_1& e_2, e_{n+1} is adjacent to e_2& e_3 and so on. Since the induced subgraph of U is complete in (W_n^f) , these nodes are mutually independent in $J(W_n^f)$ and also it can be easily verified that U is a minimal steiner dominating set. If any two successive nodes are replaced by a common neighbour in V, then the resulting set of nodes cannot be a steiner set because the removed nodes does not lie in any steiner tree of the resulting set of nodes. Thus it is not possible to choose a steiner dominating set with less number of nodes than that of U. Therefor U is a minimum steiner dominating set with 'n-1' nodes. Hence $\gamma^{fs}\left(J(W_n^f)\right) \leq n - 1$.

CONCLUSION

This article investigates steiner domination numbers of line fuzzy graphs and jump fuzzy graphs of some standard graphs such as complete bipartite graphs, wheel graphs, cycle graphs and path graphs which have extensive applications in the field of science and technology.

REFERENCES

[1] Benjier H.Arriola, "A note on some domination of fuzzy graphs'', Far East Journal of Applied Mathematics(2017), 113-123.
[2] O.T.Manjusha and M.S.Sunitha, "Total Domination in fuzzy graphs using strong arcs'', Annals

of Pure and Applied Mathematics,Vol 9, No:1, 2015, 23-33.
[3] K.Ramalakshmi,K.Palani, "On Steiner Domination number of graphs", International Journal of Mathematics Trends and Technology-Vol 4 No 2-January 2017.
[4] Samir K.Vaidya and Raksha N Mehta, "On Steiner Domination in Graphs", Malaya Journal of Matematik,Vol 6,No 381-384,2018.
[5] Samir K Vaidya and S.H.Karkar, "Steiner Domination number of some graphs", International journal of Mathematics and Scientific computing (ISSN : 2231-5330),Vol 5,No: 1,2015.
[6] M.S Sunitha and Sunil Mathew, "Fuzzy Graph Theory: A Survey", Annals of pure and applied mathematics,(2013), 92-110.
[7] Yashwant Vaishnaw and Sanjay Sharma, "Some new approaches of fuzzy graph", IOSR Journal of Mathematics,(26016), 109-112.